2 Quantitative Science

Learning Objectives

As you work through this chapter you will learn how to:

- express measurements in scientific units.
- create and use a solution map.
- convert between common American units and scientific units.
- determine the number of significant figures in a measurement.
- round off a calculated result to the proper number of significant figures.
- apply a systematic approach to solving chemistry problems.

2.1 A Classic (Coke) Experiment

If you have ever been to a party which had a large container of ice water to keep cans and bottles of beverages cold, perhaps you've noticed that some of the cans of soda sank in the water while other cans floated. In Figure 2.1 you see that an unopened can of Classic Coke placed in a large container of water sinks. Why do you think this happens? A common answer is "Coke is heavier than water". This answer is a thoughtful one but is too vague to be correct. It also overlooks the fact that the sample is a can of Coke, not just the liquid soft drink. A better explanation is that the can of Coke is heavier than the amount of water that it displaces.[1] Another way of saying this is that the can of Classic Coke sinks because it is denser than water.

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[1]This is an application of **Archimedes' principle** that states when an object is placed in a fluid (a gas or liquid) it is buoyed upward by a force equal to the weight of the fluid it displaces. If the weight of the object is greater than the weight of the displaced fluid, then the force of gravity is greater than the buoyancy force and the object will sink.

Figure 2.1 A can of Classic Coke sinks in water

In Figure 2.2 a similar situation is shown in which an unopened can of Diet Coke is placed in a container of water. Notice that this time the can floats. Based on the assessment of the Classic Coke observation, one can suggest that the can of Diet Coke floats because it is less dense than the water.

Figure 2.2 A can of Diet Coke floats in water

Thus, we have two observations and two tentative explanations. Our explanations allow us to make predictions that can be tested. This application of the scientific method is summarized below.

Observation	Hypothesis	Prediction	Test
can of Classic Coke sinks in water	can of Classic Coke is denser than water	density of can of Classic Coke will be greater than that of water	measure the density of the can of Classic Coke and of water and compare
can of Diet Coke floats in water	can of Diet Coke is less dense than water	density of can of Diet Coke will be less than that of water	measure the density of the can of Diet Coke and of water and compare

In order to carry out the proposed tests the quantity that we need to measure is **density**. Density (d) is equal to the mass (m) of an object divided by its volume (V). In mathematical terms,

$$\text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad d = \frac{m}{V} \qquad (2.1)$$

density examples

In order to get the density of an object, one can measure its mass and volume and divide the measured values. Before considering these measurements further we need to talk about units.

2.2 Scientific Units of Measure

Every quantitative measurement is associated with both a number value and a unit or units. In fact, without the units being specified a measurement doesn't tell us anything. If you are told that it takes about 5 to go from CSUN to your friend's home, you can't determine if they live right across the street from campus (5 minutes) or near San Francisco (5 hours). In this case the minutes or hours are the units that give the numerical value its meaning. The case of the Mars Climate Orbiter launched by NASA in 1998 illustrates the importance of units. Thruster data for the mission was provided by Lockheed Martin, the company that built the spacecraft, to the NASA navigation team. Lockheed was using English units (pounds) in its calculations while NASA was using and expecting values in metric units (newtons). Since the units for the thruster data were never explicitly noted, the spacecraft approached the planet's surface closer than was intended and is believed to have burned up in the Martian atmosphere. This mistake cost $328 million.

Each fundamental quantity, like mass and volume, has a scientific unit of measure associated with it. This **International System of Units** (abbreviated **SI** from the French *le Système international d'unités*) is used by scientists. The SI system involves a **base unit** of measure that is scaled up or down by multiplying it by a multiple (power) of ten. The scaling factor is designated by a prefix attached to the symbol of the base unit. For example, the base unit for length in the SI system is the **meter** (m). This can be scaled down by a factor of 1/100 by attaching the *centi-* prefix to the base unit, thus creating the unit of centimeter (cm). The name and symbol of some common SI scaling factors, along with the corresponding power of ten and the decimal equivalent, are shown in Table 2.1. Memorize each of these prefixes and its power of ten scaling factor.

Table 2.1 Common SI prefixes[2]

Prefix name and symbol	Power of ten scaling factor	Power of ten decimal equivalent
tera (T)	10^{12}	1,000,000,000,000
giga (G)	10^{9}	1,000,000,000
mega (M)	10^{6}	1,000,000
kilo (k)	10^{3}	1,000
base unit		
centi (c)	10^{-2}	0.01
milli (m)	10^{-3}	0.001
micro (μ)	10^{-6}	0.000001
nano (n)	10^{-9}	0.000000001
pico (p)	10^{-12}	0.000000000001

scale of the universe

If you commute to campus you probably refer to the distance you travel in miles rather than feet or inches because the numerical value in miles is a convenient number to use. The scaling factors in the SI system serve the same purpose - they make reference to a measured quantity more convenient. For example, if one measures a distance of 10,000 m, one can use the **scaled unit** kilometer (km) because it is a more convenient way to express this value; 10,000 m is equivalent to 10 km.

The conversion between a measurement expressed in terms of a base unit or a scaled unit (for example, m or km) to another scaled unit (such as cm), or vice-versa, represents a very fundamental unit conversion process. All of the quantitative problems in this course will involve unit conversions of some sort so it is very important to develop the skills needed to make these calculations. In fact, you already use such skills when you convert between inches and feet (12 inches is equal to 1 foot). The advantage of the

[2]All 20 SI prefixes can be found at **http://physics.nist.gov/cuu/Units/prefixes.html**.

SI system is that you need to memorize only one set of scaling factors which you can apply to all units of measure!

The simplest example of an SI unit conversion involves converting a scaled unit quantity (such as km) to the equivalent base unit value (in this case m). Simply replace the scaled unit prefix with its power of ten prefix equivalent. This is illustrated in the example below.

Example 2.1

Problem

Convert 5.8 km to meters.

Solution

Replace the scaling factor prefix (k) with 10^3. Thus, 5.8 km = 5.8 x 10^3 m.

Note: The answer above is an example of a numerical value written in **scientific notation**. See Appendix A for a detailed discussion of scientific notation and a review of doing calculations using powers of ten.

The reverse process, converting a base unit quantity to its equivalent scaled unit, for example, *m* to *μm*, may not seem as trivial. However, it is something you do rather routinely when making a conversion such as feet into inches. Suppose you are asked to convert 1.5 feet into an equivalent number of inches. Since there are 12 inches in 1 foot, you probably just multiply 1.5 by 12 to get 18 inches. In fact, this simple unit conversion involves multiplying the starting quantity by a ratio that is numerically equivalent to one (unity) so that the numerical value is not changed and that the starting unit (ft) is converted to the desired quantity (in). The ratio that is used is obtained from an equality established between the two desired units, in this case between feet and inches; 1 ft = 12 in. The formal calculation looks like this:

$$1.5\,\cancel{\text{ft}} \times \frac{12\,\text{in}}{1\,\cancel{\text{ft}}} = 18\,\text{in}$$

Notice that we multiply by *12 in* over *1 ft* so that the unit in the denominator (ft) cancels the starting unit (ft), leaving us with *inches* for the unit in the answer.

For making a conversion between an SI base unit and its equivalent scaled unit, we can build on the idea of replacing a prefix with its power of ten equivalent (notice the pattern to our approach!). The equality that is needed to create the unit ratio is easily established between a scaled unit and its base unit by starting with 1 scaled unit and replacing the prefix with the appropriate power of ten. That is, 1 μm = 1 x 10^{-6} m (Note that 1 x 10^{-6} is the same as 10^{-6}.). Since the two quantities in this equation are equal, a ratio of these quantities equals one. This ratio can be written in one of two ways:

$$\frac{1\,\mu\text{m}}{10^{-6}\,\text{m}} \quad \text{or} \quad \frac{10^{-6}\,\text{m}}{1\,\mu\text{m}}$$

Which of the above ratios you use is determined by the unit conversion you are trying to achieve. In our example, since we are starting with *m*, we want *m* to be in the denominator so that it will cancel (divide out) the starting unit and leave us with *μm*, the desired unit in the numerator. Since the conversion factor ratio is equal to one, the value of the starting amount is not altered. The role of the conversion factor is simply to change units. This process is illustrated in Example 2.2.

Example 2.2

Problem

Convert 8.7 m to micrometers (μm).

Solution

For m ➔ μm, we need a conversion factor with units of $\frac{\mu m}{m}$ which will cancel *m* and introduce *μm* into the numerator.

Since 1 μm = 10^{-6} m we use $\frac{1\,\mu m}{10^{-6}\,m}$.

Multiply 8.7 m by the unit ratio $\frac{1\,\mu m}{10^{-6}\,m}$ and cancel *m* in both numerator and denominator.

$$8.7\,\cancel{m} \times \frac{1\,\mu m}{10^{-6}\,\cancel{m}} = 8.7 \times 10^{6}\,\mu m$$

Remember that whenever writing an equality between a scaled unit and its base unit, the prefix is always replaced by its corresponding power of ten. The power of ten should never appear on the same side of the equality as its prefix (that is, 10^{-6} μm ≠ 1 m).

A more challenging problem is the conversion between two scaled units. Generally one has not memorized the equality between two scaled units so this type of unit conversion problem involves two steps. First, the initial scaled unit is converted to the base unit, and then the base unit is converted to the desired scaled unit. This stepwise process of unit conversion is sometimes referred to as a **solution map** or **dimensional analysis**. The solution map for the conversion between *km* and *mm* is:

km ➔ m ➔ mm

Example 2.3 illustrates how to set up and perform a multi-step unit conversion.

Example 2.3

Problem

Convert 4.9 cm to nanometers (nm).

Solution

First create the appropriate solution map.

cm ➔ m ➔ nm

Each step requires a conversion factor.

For cm ➔ m, we need a conversion factor with units of $\frac{\text{m}}{\text{cm}}$ which will cancel *cm* and introduce *m* into the numerator.

Since 1 cm = 10^{-2} m we use $\frac{10^{-2}\ \text{m}}{1\,\text{cm}}$.

For m ➔ nm, we need a conversion factor with units of $\frac{\text{nm}}{\text{m}}$ to cancel *m* and introduce *nm* into the numerator.

Since 1 nm = 10^{-9} m we use $\frac{1\,\text{nm}}{10^{-9}\ \text{m}}$.

Now multiply 4.9 cm by both unit ratios and cancel common terms in both numerator and denominator.

$$4.9\ \cancel{\text{cm}}\ \text{x}\ \frac{10^{-2}\ \cancel{\text{m}}}{1\ \cancel{\text{cm}}}\ \text{x}\ \frac{1\,\text{nm}}{10^{-9}\ \cancel{\text{m}}} = 4.9\,\text{x}\,10^{7}\ \text{nm}$$

Note: When dividing powers of ten, subtract the denominator exponent from the numerator exponent. See **Appendix A** for a review of calculations involving powers of ten.

Check for Understanding 2.1 **Solutions**

1. Write the solution map for the conversion of *pm* to *km*. Indicate the numerical ratio that is the conversion factor in each step.

2. Convert 233 kg to an equivalent number of micrograms.

3. Find the correct numerical value of *x*. $\frac{9.15 \times 10^7 \text{ cycles}}{\text{s}} = \frac{x \text{ cycles}}{\text{ms}}$

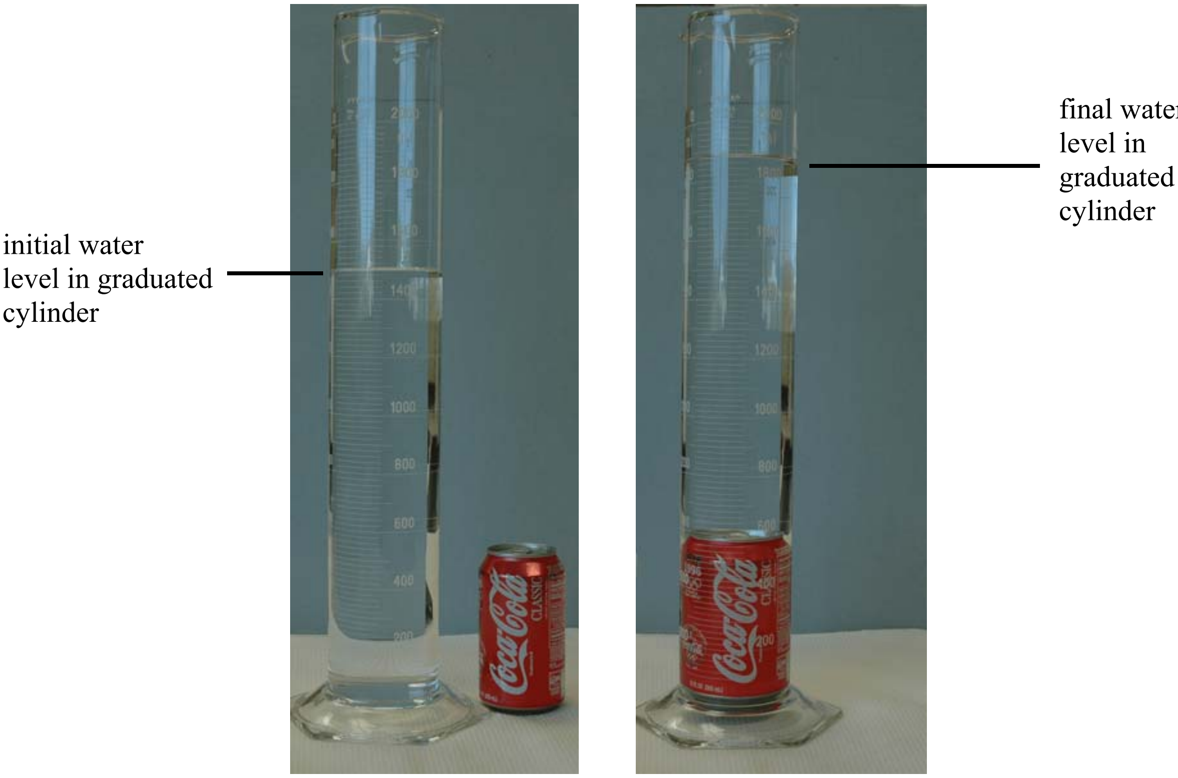
4. Write each of the following in proper scientific notation.

   a) 0.000000203 b) 12,918

2.3 Classic Coke Experiment Revisited

Now we can return to our cans of Coke and get back to testing our hypotheses. The mass of objects the size of the can of Classic Coke is usually measured in units of **grams** (g) or **kilograms** (kg) . It is very useful to have an appreciation for how these quantities correspond to familiar American units of mass. For example, a nickel (5-cent coin) weighs about 5 g and a medium-sized (2 lb) bag of brown sugar weighs close to one kilogram. Thus, you might expect a can of Coke to weigh hundreds of grams, or less than one kilogram. The mass of the can of Classic Coke and Diet Coke can be measured readily using a laboratory balance.

The volume of each can of Coke, however, is more difficult to determine because of the irregular shape of the can. Note that we are interested in the total sample volume not just the volume of liquid in the can. If the can were a regular cylinder, then one could calculate its volume from the radius (r) and height (h) of the cylinder ($V = \pi r^2 h$). For irregularly shaped objects like these cans, one can use water displacement to determine the object volume. In this approach, first the volume of a sample of water is measured using a device called a graduated cylinder (see Fig. 2.3a). The graduated cylinder has markings to indicate the volume level of the water. After the water level is recorded, the can of Classic Coke is placed in the water and allowed to sink (see Fig. 2.3b). The water level increases as the can displaces some of the water and the difference between the two levels (final volume - initial volume) is the sample volume. The same measurement is done for the Diet Coke except it must be pushed beneath the water surface so that its total volume is measured.



(a) (b)

Figure 2.3 Measuring the volume of an irregularly-shaped object by water displacement
(a) Initial water level in graduated cylinder. (b) Final water level after submerging the object.

The unit of volume commonly used in science is the **liter** (L), although it is not an official SI unit. One liter is the volume of a cube with an edge length of 10 cm. Thus, one liter is equivalent to 1000 cm^3 and is slightly larger than the American quart.[3] The volume of liquid samples in chemistry experiments is often rather small and the milliliter

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[3]Note that 1 L also equals 1000 mL, therefore 1 cubic centimeter (1 cm^3) = 1 mL.

(mL) unit is a more convenient measure. One teaspoon of liquid is about 5 mL. Therefore, one might expect the Coke can volume to be hundreds of milliliters.

Whenever you encounter quantities expressed in SI units create a mental picture of the amount involved by comparing the given quantity to a familiar reference point. The relationship between the SI unit and the U.S. unit for some common quantities is given in Table 2.2.

Table 2.2 Some useful SI unit - U.S. unit conversion factors

Quantity	Conversion Factor
distance	1 in = 2.54 cm (exactly)
mass	1 lb ≃ 454 g‡
volume	1 gal ≃ 3.78 L‡

‡This is not an exact relationship; ≃ means *approximately equal to*.

With the mass and the volume of each can of Coke, the sample density can be calculated in g/mL units. By measuring the mass of a specific volume of water, its density can also be determined. However, water[4] is a pure substance so a different approach can be used to obtain its density. A **pure substance** is matter that has a definite composition, that is, its composition is always the same. Since it has a specific composition, a pure substance has a unique chemical formula. Furthermore, since the composition of a pure substance is always the same, it has specific properties. If we did not know that the large beaker in Figure 2.1 contained water, we could measure various properties such as the boiling point, freezing point and density of the liquid and compare these values with those of water to decide if in fact this was a pure water sample. The properties of many pure substance are tabulated and can be looked up easily instead of making a measurement. We will consider pure substances in more detail in Chapter 3.

[4]In this context we are referring to water that has been extensively purified so that incredibly small amounts of impurities remain, as opposed to tap water which contains much higher and easily detected impurity levels.. The water used in a typical chemistry laboratory is called **deionized water** which has parts per billion or lower impurity levels.

If you search for the density of pure water you will discover that its density depends on the temperature of the sample. Recall that density is mass/volume. The mass of a given object does not change with temperature, however, its volume does. Most substances generally expand (volume increases) as the temperature goes up. Thus, as the temperature (and hence volume) increases one expects the density of a substance to decrease because you are dividing by a larger number. This trend can be seen from the density values for pure water measured at various temperatures in Table 2.3. Notice that the temperatures are reported in degrees Celsius (°C). The **Celsius temperature scale** is the one commonly used in the chemistry laboratory. A Celsius temperature $t_{°C}$ is related to its equivalent Fahrenheit temperature $t_{°F}$ according to:

$$t_{°C} = \frac{(t_{°F} - 32)}{1.8} \qquad (2.1)$$

Table 2.3 Density of Pure Water at Various Temperatures

Temperature (°C)	Density (g/mL)
10.0	0.9997
15.0	0.9991
20.0	0.9982
21.0	0.9980
22.0	0.9978
23.0	0.9975
24.0	0.9973
25.0	0.9970
26.0	0.9968
27.0	0.9965
28.0	0.9962
29.0	0.9959
30.0	0.9957
35.0	0.9940
40.0	0.9922

Common room temperatures of 68-72 °F correspond to 20-22 °C.[5] The temperature of the water used in the Coke can experiments was measured to be about 22 °C so a density of 0.9978 g/mL can be used for the water. The mass and volume measurements of the Coke samples along with the calculated sample densities are given below.

Data for density comparison

Sample	Mass (g)	Volume (mL)	Density (g/mL)
can of Classic Coke	388.5	390	0.9962
can of Diet Coke	375.3	390	0.9623
pure water (22 °C)	nd‡	nd‡	0.9978‡‡

‡not determined ‡‡from Table 2.3

Recall our hypotheses: the can of Classic Coke sank because it was denser than water and the can of Diet Coke floated because it was less dense than water. Now look closely at the results. The measured density of the Diet Coke sample is less than the known density of water, thus supporting our hypothesis. However, a comparison of the measured density of the Classic Coke sample with the water density does not support our hypothesis. Does this mean that we need to revise our hypothesis (and perhaps Archimedes' principle) or is there another way out of this predicament?

2.4 Significant Figures and the Quality of Measurements

The resolution to this problem is linked to the measuring process. You might suggest that the mass and volume measurements were not done carefully so perhaps the calculated densities are in error. In fact, all of the measurements were done very

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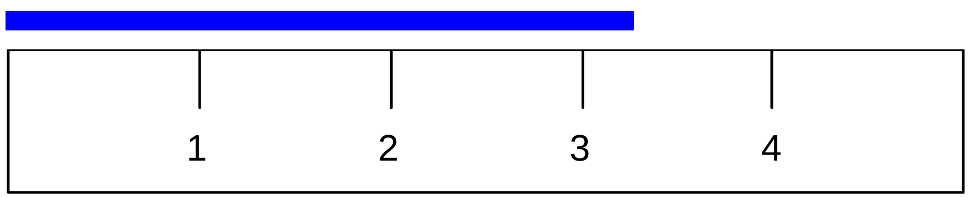
[5]You may have noticed that the temperatures plotted in Figure 1.5 (p. 25) were expressed on the **Kelvin scale** (K), the SI temperature unit. The connection between a Celsius temperature $t_{°C}$ and its equivalent Kelvin temperature t_K is given by $t_{°C} = t_K - 273.15$. Note that there is no degree symbol (°) associated with Kelvin temperatures. **Compare temperature scales.**

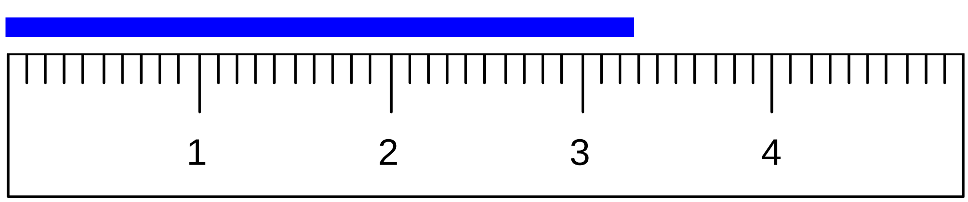
carefully and even repeated to ensure that no systematic errors were made. However, there is some limitation associated with every measurement. This limitation depends on the quality of the measuring tool. For example, suppose you wanted to know the length of the blue line below.

Perhaps your first measure is an "eyeball" estimate, say about 3 inches in length. This is a rather crude measure, after all you did not use any specific measuring tool. Now determine the length of the line using the ruler shown below. Since it is graduated in 1-inch intervals, you can quickly see that the line is a little more than 3 inches in length and you might estimate it to be 3.3 inches.



1-inch intervals

Now repeat the measurement using the ruler below. Since it is graduated in 1/10–inch intervals you can determine that the line is just slightly less than 3.3 inches and you might measure its length at 3.27 inches. Notice that the measurement becomes more certain (3 in ➔ 3.3 in ➔ 3.27 in) as the quality of your measuring tool improves. This is generally the case; of course, it assumes that all measurements are being done as carefully as possible.



1/10-inch intervals

When scientists report quantitative measurements they indicate the quality of the measuring tool(s) used, and hence the quality of their measurements, by using a convention known as **significant figures**. The term *significant figures* refers to all the digits in a measurement that are known with certainty plus one digit that is estimated by the person making the measurement. The uncertain digit is always the one furthest to the right. The last length measurement above (3.27 in) has 3 significant figures. The units and tenths place (3.2) are certain, but there is uncertainty in the hundredths place. Unless specified otherwise, it is assumed that the uncertainty in the measurement is ±1 in the rightmost digit. This means that based on the last ruler we used the actual length of the line is between 3.26 and 3.28 inches.

Note that the number of significant figures in a measurement does not depend on the units. For example, if you measure a mass as 49.8 mg (3 significant figures) and then convert it to grams (0.0498 g) it will also have 3 significant figures. This means that the leading zeros (the zeros that appear in a number before the first non-zero number) are not included in the significant figure count. What about trailing zeros (the zeros that appear after the last non-zero number)? If a mass is reported as 1.3900 g, how do we determine the number of significant figures in this measurement? The usual convention is to count trailing zeros as significant figures if there is a decimal point explicitly shown in the measurement. Thus, a reported mass of 1.3900 g has 5 significant figures. If there is no decimal point, such as in 85400 m, then do not count the trailing zeros as significant figures. Thus, 85400 m has 3 significant figures.

In summary, all digits in a measurement are significant except trailing zeros in numbers without a decimal point and all leading zeros.

There are two other important issues regarding significant figures. First, exact numbers, such as the number of objects (12) in a dozen, have an unlimited number of significant figures. Such numbers can be written with as many trailing zeros as you wish (e.g., 12.0000...). The conversion factor in Table 2.2 between inches and centimeters is an exact number (2.54 cm/in) so you are not limited to 3 significant figures. The same is true for relationships like 1 mm = 10^{-3} m; these numbers are exact. Often it is clear from the context of a calculation whether a number is exact or not.

Finally, when measurements are written in proper scientific notation all of the digits are significant, thus there is no ambiguity about the number of significant figures. For example, a mass of 0.060190 g has 5 significant figures and when it is written in scientific notation as 6.0190×10^{-2} g it still has 5 significant figures. The use of scientific notation also allows one to handle situations like writing a quantity such as 400 m with 2 significant figures (4.0×10^{2} m). In decimal form this would have either 3 significant figures (400. m) or just one (400 m).

Check for Understanding 2.2 **Solutions**

1. How many significant figures are in each of the following measurements?

   a) 588.0 kg   b) 12,000 miles   c) 0.00700100 s

2. Write each of the quantities in question 1 in proper scientific notation.

2.5 Significant Figures in Calculated Results

Most quantitative results that scientists report (like the density values for our Coke samples) are calculated from a number of direct measurements (like the mass and volume values for our Coke samples). In order for these calculated results to properly reflect the quality of the measuring tools that acquired the data used in the calculations, they must be consistent with the significant figures obtained in the direct measurements. It is not appropriate to simply report as many figures as are in your calculator display. Instead, the calculated result is rounded off[6] to a consistent number of significant figures. So how do you determine the appropriate number of significant figures to report? There are guidelines that depend on the specific calculations used to obtain the result.

[6]Rounding off refers to the process of dropping all digits to the right of the last retained digit. There are conventions used to decide how to round off a result depending upon the value of the digit immediately to the right of the last retained digit (see Example 2.4).

Calculations involving only multiplication and/or division

Guideline: For multiplication and/or division calculations, the result is rounded off to as many significant figures as are found in the measurement with the fewer (or fewest) number of significant figures.

This makes sense because we can't "know" a calculated result better than we "know" the measurement made with the least precise tool. This guideline is illustrated in Example 2.4.

Example 2.4

Problem

Perform the following calculation and round off the result to the proper number of significant figures. The units of these measurements have been omitted for clarity.

$$\frac{14 \times 5.18}{96.1} =$$

Solution

The calculated result is $\frac{14 \times 5.18}{96.1} = 0.75463...$

Note the number of significant figures in each measurement in this calculation.

14 (2 sig. fig.) 5.18 (3 sig. fig.) 96.1 (3 sig. fig.)

Since 2 significant figures is the fewest number in any measurement, the result is rounded off to 2 significant figures.

0.75463... ➔ 0.75

There are also guidelines to follow when you need to round off the result. When rounding off, if the first digit to be dropped is less than 5, the preceding digit stays the same (as in the example above). If it is greater than 5, the preceding digit is increased by 1. In the unusual case that it is exactly 5 (50000...), the preceding digit stays the same if it is even and is increased by 1 if it is odd.

<u>Calculations involving only addition and/or subtraction</u>

Guideline: For calculations involving only addition and/or subtraction, the result is rounded off to the largest place (e.g,, ones, tenths, hundredths) with uncertainty.

This guideline is illustrated in Example 2.5.

Example 2.5

Problem

Perform the following calculation and round off the result to the proper number of significant figures. The units of these measurements have been omitted for clarity.

$$\begin{array}{r} 12.46 \\ +\ \ 9.3 \\ \hline \end{array}$$

Solution

The calculated result is

$$\begin{array}{r} 12.46 \\ +\ \ 9.3 \\ \hline 21.76 \end{array} \rightarrow 21.8$$

Note that the uncertain digit (in red) in the first measurement is in the hundredths place while the uncertain digit in the second measurement is in the tenths place. Thus, the sum is rounded off to the (larger) tenths place.

Check for Understanding 2.3 **Solutions**

1. For each of the following, calculate the result and round it off to the proper number of significant figures. Units have been omitted for clarity.

   a) 28.4 ÷ 0.0091 b) 63.2 - 61.04

When results are calculated from a mix of addition/subtraction and multiplication/division operations, one must apply the appropriate guideline to each intermediate step in the calculation to determine the proper number of significant figures associated with the intermediate result. The significant figure count in the intermediate result should be noted, however, intermediate results should not be rounded off. Such "mixed" calculations will not be emphasized in this course.

Now we can apply the concept of significant figures to the calculated densities of the cans of Coke.

Sample	Mass (g)	Volume (mL)	Density (g/mL)
can of Classic Coke	388.5 (4 sig. fig.)	390 (2 sig. fig.)	0.9962 ➔ 1.0 (2 sig. fig.)
can of Diet Coke	375.3 (4 sig. fig.)	390 (2 sig. fig.)	0.9623 ➔ 0.96 (2 sig. fig.)
pure water (22 °C)	-----	-----	0.9978

Notice that the mass measurement for each can has 4 significant figures. How about the volume measurement? Since there is no decimal point in the volume measurement, the trailing zero is not significant and each volume measure has only 2 significant figures (this means that the can volume was only measured to the nearest 10 mL). Thus, when the mass is divided by the volume the resulting density should be rounded off to 2 significant figures. When you do this the measured density of the can of Diet Coke is less than that of water (as before), however, now the measured density of the can of Classic Coke is greater than that of water, which is consistent with our hypothesis. Our previous calculations had ignored the limitations of the measuring tools and had produced an erroneous result for the density of the Classic Coke sample. Note that the density listed for water has 4 significant figures. This reflects the high quality of the instruments used to measure the water density.

In summary, the use of significant figures in reported quantitative results informs the reader about the quality of the measurements. A larger number of significant figures always indicates more precise measurements.

2.6 Developing Problem-Solving Strategies

Most of the exercises so far have involved very direct questions. Often you will be confronted with problems that contain much information that must be organized in order to understand the question and develop a solution. Consider the problem below.

Problem: You are interested in supporting a 20-gallon aquarium on a small table, but you are not sure if the table will be able to support the mass of the full aquarium. Approximately how many pounds do you expect the full aquarium to weigh?

The first step in creating a solution for this problem is to clearly identify what it is that you must determine. This may require you to re-read the entire question very carefully, and then note what you are looking for and what information you have to get there. For problems that involve calculations you should focus on the units required in the answer. In the problem above you want to determine the mass, in pounds, of a filled aquarium, so pounds represents the end point of your solution map. Next identify what information is provided, especially with regard to units of measure. The only quantitative information given is the aquarium volume of 20 gallons. Thus, gallons represents the beginning point of your solution map which now looks like this:

gal water (in aquarium) ➔ lb filled aquarium

Notice that the problem asks for an approximate mass for the filled aquarium. Since the water filling is likely to constitute the majority of the mass, you can simplify the solution map above by ignoring the mass of the aquarium (and other materials like the stones used to cover the bottom) and calculate the mass of the water only. The solution map then becomes:

gal water (in aquarium) ➔ lb water (in aquarium)

This simplified solution map clearly indicates that the problem requires a conversion from the volume of a substance (water) to its mass. This suggests to us that the conversion factor that is needed is the density of water; remember density is the quantity that relates mass and volume (d = m/V). At this point you might search the Internet for the density of water in units of lb/gal, however, this is time consuming and is not an option on quizzes and exams. Density values for water are readily available in this chapter in g/mL so you should now think about unit conversion steps that utilize this information along with other conversion factors with which you are familiar. For example, with the density unit of g/mL you need a volume unit of milliliters or even liters. The connection between gallons and liters is 1 gal ≃3.78 L. Also, when you use a g/mL density and get the mass of the water from its volume the mass will have units of grams so you will need to convert between grams and pounds (recall that 1 lb ≃ 454 g). Putting all of these ideas together results in the following solution map.

gal water ➔ L water ➔ mL water ➔ g water ➔ lb water

This four-step solution map requires 4 conversion factors. The good news is that you are familiar with all four factors so you now can set up the mathematical solution. If you focus on the unit conversions, the setup looks like this:

$$\cancel{\text{gal water}} \times \frac{\cancel{\text{L water}}}{\cancel{\text{gal water}}} \times \frac{\cancel{\text{mL water}}}{\cancel{\text{L water}}} \times \frac{\cancel{\text{g water}}}{\cancel{\text{mL water}}} \times \frac{\text{lb water}}{\cancel{\text{g water}}} = \text{lb water}$$

Notice how the units in the properly designed solution map cancel to give the units desired in the numerator of the answer. All that remains is to insert the numerical values into the conversion factors and do the math. Remember that each step involves a conversion factor coming from an equality between the units in a particular ratio. The ones needed for this problem are:

1 gal ≃ 3.78 L   1 mL = 10^{-3} L   1.0 g water ≃ 1 mL[7]   1 lb ≃ 454 g

[7]From Table 2.3 you see that the density of water at room temperature is about 1.0 g/mL.

When you insert the numerical values into the solution map and do the calculations the result is:

$$20\ \text{gal water} \times \frac{3.78\ \text{L water}}{1\ \text{gal water}} \times \frac{1\ \text{mL water}}{10^{-3}\ \text{L water}} \times \frac{1.0\ \text{g water}}{1\ \text{mL water}} \times \frac{1\ \text{lb water}}{454\ \text{g water}} = 170\ \text{lb water}$$

It may be surprising to find that the filled aquarium weighs as much as some of your classmates.[8] This value should give you a sense of whether or not your table will support this mass.

The solution to this problem may seem overly involved (it took almost 2 pages!), but remember so much of this was a description of the steps involved. With practice you will find that creating the necessary solution map can be done easily. Here is a summary of the important aspects to organizing the information for a given problem and setting up the solution to problems involving calculations.

1. Read the entire problem very carefully, making note of key terms and units.
2. Identify the question to be answered and make special note of the units for the answer.
3. Look at all the information that is given, including units for numerical values.
4. Think about relationships that connect the given information to the question and its plausible answer and create your solution map, identifying each step.
5. Identify all the conversion factors needed for your solution map.
6. Confirm that the relevant units cancel to give you the desired units for the answer.
7. Do the calculations carefully. Think about whether or not your answer makes sense.

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[8] The answer is shown with 2 significant figures based on the density value used. The 20 gallons is assumed to be an exact number.

Some questions that you will encounter will require you to provide a brief explanation. These "concept" type problems are designed to check your understanding of core ideas. Many of the same ideas used for the calculation problems are helpful in responding to such questions. Consider the question below.

Problem: Why is it important to understand how to interpret significant figures in reported scientific measurements?

This "why" question requires a short explanation. First, you must read the question carefully because almost all of the words in this question are important to understanding how to respond. It is very helpful to rephrase the question in your own words. For example, this question is asking about the value of significant figures to a reader of a scientific report. In order to address this question one must have a solid understanding of the key term "significant figures". In this case, the definition alone ("all of the digits in a measurement that are certain, plus one uncertain digit") is not enough to answer this "why" question. It is important to appreciate the implications of the significant figures convention, namely that it reflects the quality of the tools used for the measurement, and therefore the quality of the measurement. A acceptable answer to this question might be:

> "The number of significant figures provides information about the quality of the tools used to make the measurement and hence the errors associated with the experiment. This is especially important to someone who wishes to reproduce the experimental results."

Consider this slightly different problem.

Problem: Define the term significant figures.

The correct definition is noted in the paragraph above. A response such as: "significant figures are all those digits that are important" is insufficient; it fails to identify what is meant by "significant", and it does not connect the term to a measurement.

Keep the following points in mind as you answer "concept" questions.

1. Read the entire question very carefully, underlining key terms.

2. Rephrase the question in your own words and identify the problem to be solved.

3. The correct answer never involves just the repetition of the information provided in the question.

4. Make sure that your answer is clearly stated and is complete. Your instructor cannot assume that you meant to say something that you did not indicate in your answer.

You might encounter calculation problems or concept questions formatted as multiple-choice questions. There may be only one correct choice or there can be two or more correct choices. Your success in answering such questions correctly will improve if you first eliminate choices that are not correct. For example, recall problem 2 in the Check for Understanding 1.1 on page 15. Let's look at this again. It is really a set of four multiple-choice questions. Your choices for each part are *law*, *theory*, *observation* or *none of these*. Consider the problem in part (b): *Flammable materials always contain oxygen.* This cannot be a *theory* because it does not provide an *explanation* for flammability. If this is to be an *observation* then, because of the term *always*, there must not be any exception to the statement. An exception would also rule out this statement being a *scientific law*. So can you think of any substance that is flammable that does not contain oxygen? Perhaps you know from experience that hydrogen gas is flammable. Since this does not contain oxygen these two choices are eliminated and you are left with *none of these* as the correct answer. Identifying wrong multiple-choice answers is one key to success with such questions.

Finally, no matter what type of homework, quiz or exam question you face, you should always ask your instructor to clarify the question if you are not sure of what is being asked. It is better to do this than to take the chance of misunderstanding the question.

Chapter 2 Keywords

Glossary

Archimedes' principle	scientific notation	pure substance
density	solution map	deionized water
SI units	dimensional analysis	Celsius temperature scale
base unit	gram	Kelvin temperature scale
meter	kilogram	significant figures
scaled unit	liter	

Supplementary Chapter 2 *Check for Understanding* questions

Chapter 2 Exercises

Answers

1. Write each of the following in proper scientific notation.

   a) 0.06810 b) 35,140 c) 287×10^{-3}

2. How many significant figures are in each of the following measurements?

   a) 400.010 cm b) 0.003310 g c) 500 lb d) 8.8×10^{-7} m

3. Round off each of the following measurements to 3 significant figures.

   a) 3.004 g b) 0.0006849 m c) 21,457 mL d) 89,000 s

4. For each of the following, calculate the result and round it off to the proper number of significant figures. The units of these measurements have been omitted for clarity.

   a) $2.3 \times 10^{-2} + 8.1 \times 10^{-2}$ =

   b) 9.14 / 5800 =

   c) 0.030 x 0.300 x 0.003 =

   d) 341.7 - 22 =

5. What is the average mass of three objects whose individual masses are 10.3 g, 9.234 g and 8.35 g?

6, What is the difference between a base unit and a scaled unit in the SI system?

7. What is the solution map for each of the following unit conversions?

a) ps ➔ Ms   b) g/mL ➔ kg/nL

8. Supply the correct number, in proper scientific notation, for each of the following. Show all your work.

a) 126 ms = __________ ns

b) 0.32 µg = __________ kg

c) 14.33 in = __________ m

9. What is the conversion factor between *kg* and *cg*?

10. Which of the following is the longest time interval?

A. 16 s   B. 4000 ms   C. 2.6×10^{-4} ks   D. 3.2×10^{7} ns

11. A certain brand of gasoline has a density of 0.67 g/mL. If a car has a gas tank capacity of 16 gallons, how many pounds of gasoline are in a full tank?

12. Which of the following is the highest temperature?

A. 28 °F   B. 249 K   C. 6.1 °C

13. The density of a sample was determined by water displacement. The initial water level was 23.4 mL. After adding the 7.59-g sample, the level increased to 27.7 mL. Calculate the sample density to the proper number of significant figures.

14. The density of chloroform is 1.48 g/mL. What is the volume in liters of 6.5 kg of chloroform?

15. A certain container weighs an unknown number of grams, which is 8 more grams than its lid. Write a mathematical expression for the combined mass of the container and lid.

16. If you are advised by your doctor to take 2 teaspoons of a medication 3 times a day for 10 days, what is the minimum volume of medication (in mL) you will need?